Performance Evaluation of the Radon Transformation in Limited Angle Radar Tomography

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Abstract

The Radon Transformation is an image processing algorithm widely used in the field of tomography. Limited angle tomography takes only a partial view of an object while conventional tomography images the target from all angles. This paper examines how effective the Radon Transformation is at reassembling images when presented with less than ideal limited angle tomographic data. In the first part of this experiment, a simulation is conducted in MATLAB where a pre-established image (Logan-Shepp phantom) is broken down and reassembled using this algorithm. In the second part, and experimental radar tomography station is built to collect real time tomographic data. The Radon Transformation is then used in an attempt to reassemble this back projection data into a cross sectional image. The effectiveness of this algorithm in these situations will be determined by the quality of the final images.

1 – Introduction

Radar imaging systems are widely studied by both engineers and scientists due to their ability to accurately reproduce images. In particular, the field of radar tomography is of great interest because these systems are capable of reproducing two, or even three-dimensional images of objects. Radar tomography's benefits have led to its adoption in several scientific fields. For instance, medical practitioners could use it for imaging the body without the use of ionizing radiation, whereas geoscientists map the subterranean world with ground penetrating radar.

The field of radar imaging has been around for at least half a century. Early radar systems were used to detect large ships on the ocean or cloud density for meteorological data. These early systems were able to detect the distance and relative size of an object, but lacked detail. Radar tomography expands on early radar detection by integrating its basic methods with computational software such as MATLAB or Mathematica. With today's computing power, which continues to become more accessible, researchers are able to add much more detail into images.

2.1 - Basic Radar Systems

Before examining how three dimensional radar tomography is implemented, it is necessary to first understand how one and two-dimensional imaging works. An electromagnetic wave (typically 100MHZ-100GHZ) is sent out from a transmitter (TX) while a receiver (RX) listens for a reflected wave. The distance of the target is computed with a basic calculation that compares the time it takes the wave to reflect back off the target to the receiver. A radar echo resembles an echo heard by the human ear. Its functions are limited to determining the relative size, distance, and location of an object. More complex imaging systems such as synthetic aperture radar (SAR) and radar tomography differ from a radar echo because they send out a series of waves that impact the target. When these systems are combined with computational software, they are able to not only look for shifts in time, but also in frequency, polarization, cohesion, and phase.

2.2 – Tomography

The field of tomography is concerned with reconstructing images of objects. This can be done with several types of electromagnetic radiation including, though not limited to radar, visible, and x-ray. A conventional tomographic station has a transmitter and receiver placed opposite of one another along a radial axis. A network analyzer sends out an incident waveform and listens for the received waveform. The received waveform will be distorted by its interaction with the object. This is observed as changes in frequency and phase. The TX/RX pair is moved by a small increment and another waveform is sent out. This process is repeated several times to produce a collection of one-dimensional echoes (Tseng and Chu). The mathematical result of this process is a $[1 \times n]$ matrix where n represents the number of one-dimensional echoes. Sweeping θ for 180 degrees will result in a complete set of back projection data for the object. Integrating this collection of echoes and performing an image reconstruction algorithm results in the reconstruction of an object in the form of a two-dimensional cross section (Knaell).

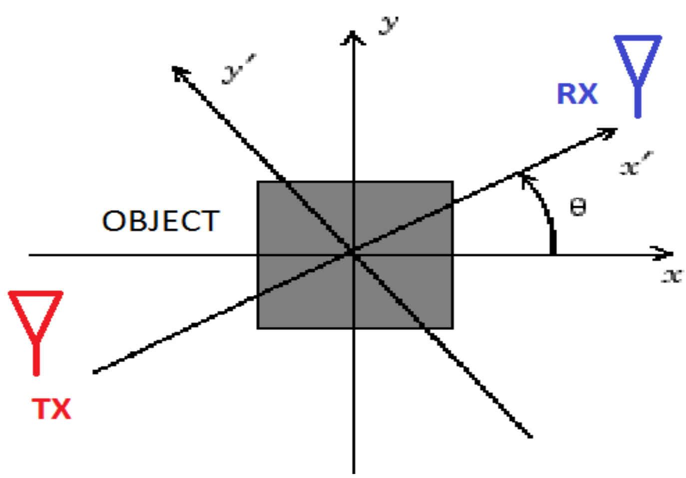


Figure 1: Depiction of a conventional tomographic approach

2.3 – Limited Angle Tomography

This paper examines limited angle tomography in particular. In a conventional tomography station, the TX/RX pair is rotated around 180 degrees around an object. This method illuminates the entire object and provides a very accurate reconstruction. However, this approach also severely limits the capabilities of such systems by requiring large, rotating gantries for imaging larger objects. Limited angle tomography differs from conventional tomography because the TX/RX pair is not rotated a full 180 degrees around a target. Such a system is capable of imaging large objects or even taking partial images that would otherwise be impossible. One drawback of limited angle tomography is that it obviously does not illuminate the entire target. This may yield less than ideal images when compared to a conventional approach.

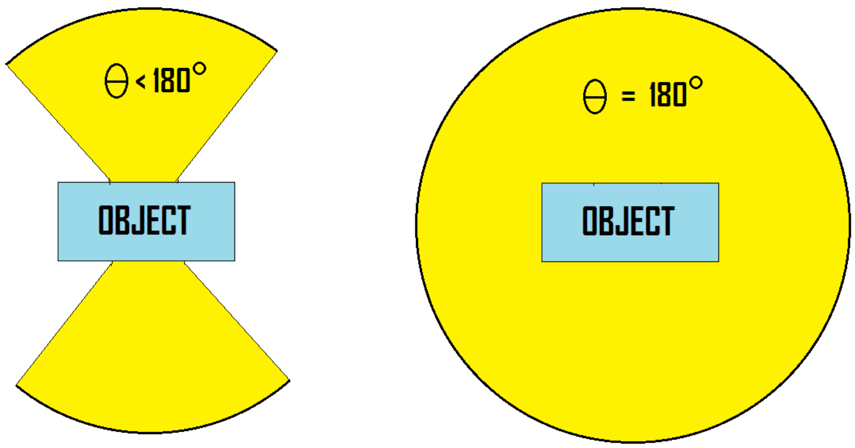


Figure 2: Range of illumination (θ) for limited angle vs. conventional tomography

2.4 –Radon Transformation

The Radon transformation is used in image processing for reconstructing images. While the mathematics of the Radon transformation are beyond the scope of this article, it is similar to the Fourier transform. When a tomography station takes an image, it stores the data as a back projection sinogram. The Fourier slice theorem states that if a sufficient number of these back projections are taken along a full 180 degrees, the object can be reconstructed perfectly (Knaell). At less than 180 degrees, however, the image clarity begins to degrade. This is due to a growing null space in the back projection data. When the null space reaches a critical size, the Radon transformation will be ineffectual at reassembling the image. Recent research has shown that it may be possible to reconstruct image with a very high degree of clarity using the Radon transformation in conjunction with interpolation and filtering algorithms (Xue, et. al.). This paper investigates the effectiveness of the Radon transformation in reproducing images without the addition of interpolation or filtering algorithms.

2.5 – Logan-Shepp Phantom

The Logan-Shepp head phantom is widely used in the field of tomography and image processing. The phantom contains several contrasted regions that replicate conditions encountered in experimental imaging. Specifically, it was developed to test image reconstructions algorithms for use in the field of medical tomography.

Figure 3: The Logan-Shepp phantom reconstructed perfectly

2.5 – Higher Order Tomography

The prime result of a two-dimensional tomographic reconstruction is a detailed cross section of an object. Furthermore, a three-dimensional image can be produced by integrating several cross sections together. Current three-dimensional imaging techniques require the transmitter or receiver be moved in two dimensions. As explained previously, the transmitter is moved along the *x*-axis to create a single two-dimensional cross section. The transmitter is then moved along the *y*-axis and the process is repeated (Knaell). This approach creates a $[m \times n]$ matrix where *m* and *n* represent the number of one-dimensional echoes in the *x* and *y* directions, respectively. Given that creating a two-dimensional cross section requires much more computing power than a one-dimensional image, developing three-dimensional images demands an extraordinary amount of computing resources.

2.6 – Frequency Considerations

When examining various objects, it is also necessary to pre-determine the relative size (mountain, warehouse, car, suitcase, etc.) The resolution of a radar system is proportional the frequency used. This can range anywhere from around 100MHz to upwards of 100GHz. The advantage of using higher frequencies is that higher resolutions can be obtained (Gilmore, et. al). On the other hand, lower frequency bands can be used to peer through foliage and walls, but lack the resolution and range of high frequency transmissions. It is also important to note that objects have different permittivity constants depending on the frequency of incident wave (Zanoon et. al.). This explains why a brick wall is opaque to the human eye at 500THz, but nearly transparent at 500MHz (Verity and Gavrilav). The same principle applies to radar imaging. For three-dimensional tomography, a band of frequencies in the range of 2GHz-10GHz have been shown to be effective at imaging objects on the scale of a few meters (Gilmore et. al). This could also be accomplished using an ultra-wideband approach.

3.1 – Experimental Station

Tomography requires that either the object or antenna must be moved to generate valid data. Conventional tomography stations involve rotating either the object or the TX/RX pair a full 180 degrees. This experiment employs limited angle tomography which allows for imaging of larger objects. In this design, the transmitter is moved along the horizontal axis for a distance of 1 meter. The object to be imaged is placed in front of the antenna pair at a specified distance. When an object is present, the incident wave is reflected by the target and picked up by the receiver. The advantage of this design is its flexibility and ease of use.

To reliably move the transmitter, it is attached to a linear actuator. This linear actuator is controlled by software, allowing it to be moved in precise increments. To move the transmitter, a script in MATLAB communicates with a National Instruments data acquisition box (DAQ). The DAQ then outputs a high signal on one of two lines to signal the linear actuator to move left or right. The DAQ alone does not enough current to drive the linear actuator, so a simple relay circuit is used between the DAQ and the linear actuator. This design can be easily expanded to three-dimensional tomography by adding another linear actuator along the vertical axis.

An Agilent network analyzer (NA) is used to send out radar waves, measure the S-parameters of the network, and port the data out for processing. The S-parameters include complex values that represent the amplitude and phase components of the reflected wave. The bandwidth of the network analyzer is set to collect data over a range of frequencies between 8GHZ and 12GHz. This produces resolutions on the order of centimeters. Ideally, a higher frequency would result in a higher resolution, but equipment capable of operating at these speeds is rare and expensive. Additionally, low noise amplifiers are attached to both the transmitter and receiver to boost the received signal strength to nearly -10dBm.

The data is collect as a $[m \times n]$ matrix where m represents the number of increments that the transmitter is moved and n represents the number of discrete frequencies. Increasing these numbers yields a higher quality image, but requires more time. A separate MATLAB script processes this data into an image by utilizing the Radon transformation.

3.2 – Image Processing Script

The image processing script is responsible for both processing the raw experimental data into an image and for the simulations. In the simulations, the phantom is first broken down into its constituent data which replicates the matrix of reflection or transmission coefficients gathered by a network analyzer. The inverse Radon transformation then reassembles the data into a final image. The quality of this image depends on both the range of illumination (θ) and the number of radial projections taken. The images were reconstructed with views ranging from 30 degrees to a full 180 degrees.

4.1 – Simulation Results

As explained previously, the image processing script was first tested with a pre-established image in the form of the Logan-Shepp head phantom. The figure below shows the results of reconstruction by varying the range of illumination.

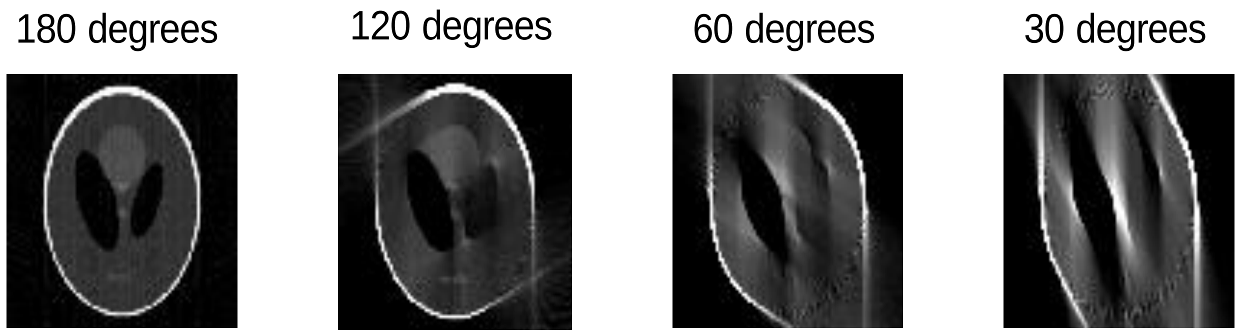


Figure 4: Results of image reconstruction based on varying ranges for θ

At a full 180 degrees, the image is reconstructed almost perfectly. The dark and light ellipses inside are easily seen and discerned. The only difference from the original image is observed in the outer white ellipse. This structure is slightly pixilated from the reconstruction process. The lack of artifacts also shows the number of projections is set sufficiently high.

At 120 degrees of illumination, the image is somewhat distorted along the corners. This is still a wide enough angle that the internal features can be observed, if not as clearly. With the addition of interpolation algorithms that are beyond the scope of this paper, it may be possible to reconstruct the image.

At 60 degrees of illumination, the outer distortions in the image are very evident. This is caused by the growing null space in the matrix where the target is not illuminated. The internal features are also skewed and blurred. The two dark ellipses in the center are still visible and retain most of their shape. The grey circles on the top and bottom, however, are beyond recognition.

At 30 degrees of illumination, the object is heavily distorted and bears little similarity to the original image. The two dark ellipses are distorted and the other grey ellipses have vanished completely.

In the experimental tomography station, the object is placed at distances of 1, 3, and 6 meters from the target while the transmitter is swept for a distance of 1 meter. Again, this was simulated through MATLAB using the Shepp-Logan phantom. The figure below shows the results of this.

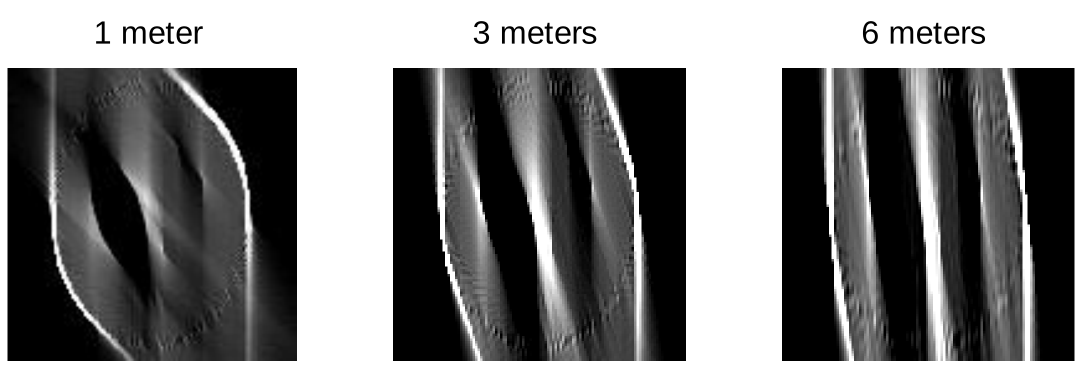
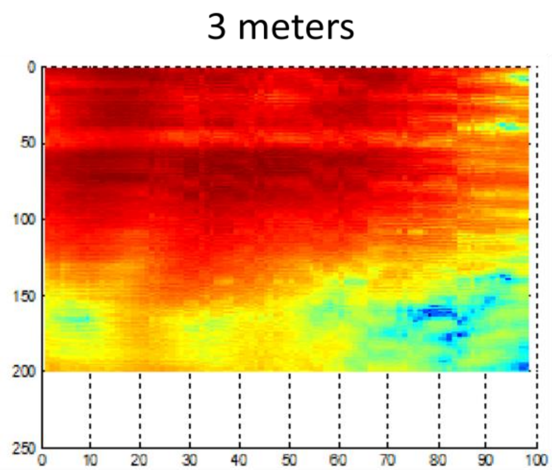


Figure 3: Results of image reconstruction at simulated distances

At a simulated distance of 1 meter, the object is heavily distorted, but some basic structures are still visible, most notably the two dark ellipses in the center. At distances of 3 meters and beyond, the object is very heavily distorted and does not display much resemblance to the original image. Based on these results, the Radon transformation alone is not effective at reproducing images at very limited ranges of illumination.

4.2 – Experimental Results

The experimental tomography station was built to collect data for later processing. The plots below show the raw range data of a human target at a distance of 1 and 3 meters, respectively.



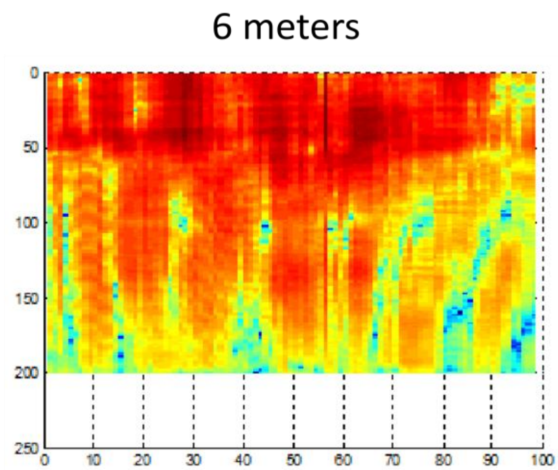


Figure 6: Range data collected by the experimental station

At a distance of 3 meters, the dark red image confirms that the station works as intended. At this close range, individual features blur together. This could also be due to unwanted antenna coupling between the transmitter and receiver. At a distance of 6 meters, we begin to see some detail and separation of features. At a distance farther than this, the features would begin to blur together again. While these images confirm the tomographic station can collect range data, it is not detailed enough to use for an accurate reconstruction.

5 – Discussion of Results

It has been shown that as the range of illumination decreases, the null space of the matrix increases. This leaves large areas of the back projection sinogram blank, and results in decreased image quality. Based on the simulation results, the Radon transformation alone is not adequate for processing limited angle tomographic data. Given sufficiently large angles though, ($\theta > 120$ degrees) the Radon transformation produces relatively accurate reconstructions. The edges of the target blur around the corners, but the interior features are still easily seen and contrasted. At smaller angles, the image distorts heavily and the internal features become nearly indistinguishable. These limited angle images may provide clues to the internal structure of a target, but cannot be relied on for high resolution images. Using the Radon transformation in conjunction with filtering and interpolation algorithms may increase the image clarity at these very limited angles.

The experimental station functions as it was intended. The range plots provide evidence that this type of station can be used for imaging. Unfortunately, this data is not clean enough to use for tomographic reconstruction. Modifying this station by placing the receiver across from the transmitter or using a larger bandwidth may or may not provide cleaner data to use for reconstruction.

6 – References

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